A new discrete optimization procedure for geometrically nonlinear space trusses

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1. Abstract
This paper introduces a new branch-and-bound type method, for discrete minimal weight design of geometrically nonlinear truss structure subject to constraints on member stresses and nodal displacements. The discrete optimization problem is formulated as a tree search problem. Each node of the searching tree is characterized by the weight and the relaxed weight of the structure, the current value of the unfeasibility penalty function, and the set of the cross-sectional areas. The initial unfeasible node of the searching tree is obtained by assigning to all members the smallest cross-sectional area from the given catalog. A child node is created from its parent node by changing exactly one cross-sectional area to the next larger one in the catalog. The algorithm maintains an upper bound (initially infinite), the minimal weight of any feasible design found so far. Any node generated can be immediately discarded if its weight or its relaxed weight is greater than or equals to the current upper bound value. The proposed method involves an exterior point method to determine the relaxed solution of the design problem.

2. Keywords
Discrete, Nonlinear, Truss, Design, Optimization, Multidisciplinary

3. Problem formulation
The geometrically nonlinear truss structure can be formulated as a large displacement model, using a total Lagrangian representation. The length of the truss member after deformation is given by the projection of the undeformed length onto the coordinate axes and the nodal displacements.
The total potential energy function of the global truss structure can be formulated in the following way:

$$V(u_i, a_j) = \frac{E}{2}\sum_{j=1}^{e}\frac{a_j}{l_j^0}\left(l_j^0 - l_j\right)^2 - p_i u_i$$
$$i = 1,2,\ldots,n \qquad j = 1,2,\ldots,e\,, \tag{1}$$

where u_i is the nodal displacement vector, a_j the vector of the member cross-section areas, p_i the external load vector at the nodal points, l_j^0 the undeformed length of truss member, l_j the length of the truss member after deformation, and E is the elastic modulus of the material.

The discrete optimization problem is discussed in terms of above defined structural model. The design variables a_j are selected from a discrete set of the predetermined $a_j \in A = \{A_1, A_2, \ldots, A_J\}$ cross-sectional areas, such that minimize the total weight of the structure

$$\rho l_j^0 a_j \rightarrow min! \tag{2}$$

subject to the

$$V_{,i}(u_i(a_j))\Big|^{a_j} = 0\,,$$
$$\left|u_i(a_j)\right| \le \bar{u}\,,$$
$$\left|s_j(u_i(a_j))\right| \le \bar{s}\,,$$
$$i = 1,2,\ldots,n \qquad j = 1,2,\ldots,e \tag{3}$$

equality constraint in form of equilibrium equations, and the stress and displacement inequality constraints.

4. The tree search problem
The discrete optimization problem can be formulated as a tree search problem. The catalog values $a_j \in A = \{A_1, A_2, \ldots, A_J\}$ of cross-sectional areas are sorted in ascending order: $A_1 < A_2 < \ldots < A_J$. The searching domain for every element is defined by the set of the lower and upper cross-sectional indices: $\{LB_j, UB_j\}$, $j = 1,2,\ldots,e$. The algorithm provides *global optimal* solution, if $\{LB_j = 1, UB_j = J\}$, $j = 1,2,\ldots,e$. The algorithm provides a local optimal solution, if we define the set of the lower and upper indices for every element by decreasing and increasing the relaxed solution to the closest discrete neighbor. We note that in this case, the efficiency of the algorithm highly depends on the *quality of the relaxed solution*.

During the optimization process, it is supposed that the structure constructed from the largest elements is feasible, i.e. the constraint functions are fulfilled. Denote the weight of this structure W^u and $I_j^u = UB_j$ the vector of cross-sectional indices. If the structure, constructed from the smallest elements, is feasible, then this structure will be the optimal solution of the discrete minimal weight design problem. Otherwise - starting from this initial structure - a searching tree will be built up. A child node of the searching tree is created from its parent by changing exactly one cross-sectional area to its immediate successor in the catalog.
The method maintains the lowest weight of any feasible solution found so far. Any node generated can be immediately discarded if its weight or its relaxed weight is greater than or equals to the current upper bound value W^u.
The nodes of the searching tree $N^0, N^1, ..., N^k ...$ are characterized by the set of

$$N^k = \left\{I_j^k, W^k, P^k, R^k, S^k\right\}, \tag{4}$$

where:

$I_j^{(k)}$ the vector of the cross-sectional indices at the k^{th} node of the searching tree,

$W^{(k)}$ the weight of the structure at the k^{th} node of the searching tree,

$P^{(k)}$ the actual value of the penalty function at the k^{th} node, where:

$$P^k = \sqrt{\sum_{i=1}^{n} \max\left(|u_i| - \bar{u}, 0\right)^2 + \sum_{j=1}^{e} \max\left(s_j - \bar{s}, 0\right)^2 + \sum_{j=1}^{e} \max\left(\underline{s} - s_j, 0\right)^2}\ , \tag{5}$$

$R^{(k)}$ the minimal relaxed additional weight, that is needed to obtain a feasible structure from the given state,

$S^{(k)}$ the status variable of the k^{th} node, where

$S^{(k)} = 0,$ if the k^{th} node is an expandable, and $S^{(k)} = 1,$ if the k^{th} node is an expanded one.

5. The stages of the searching algorithm

The algorithm consists of a series of tree expanding steps. Let sn be the number of nodes of searching tree at the *starting* of a step, and let fn be the number of nodes of searching tree at the *finishing* of a step.

Step 1. Initialize the counter $sn = 0$ and the set $N^0 = \left\{I_j^0 = LB_j, W^0, P^0, R^0, S^0 = 0\right\}$, where: $P^0 > 0$.

Step 2. Let $fn = sn$ and select the most promising node bn using the following searching conditions:

$$\begin{gathered} P^{bn} \to \min, \\ W^{bn} < W^u, \\ R^{bn} < W^u \\ S^{bn} = 0, \\ bn \in \{0, 1, ..., sn\} \end{gathered} \tag{6}$$

If the search process fails the algorithm terminates. The solution of the problem is $\left\{W^u, I_j^u\right\}$.

If the search process results alternative nodes, then the node with smallest serial number is selected. $S^{bn} = 1$.

Select the first cross-sectional index from index set I_j^{bn} which does not reach the maximal index. If there is no such an index, then go to Step 2.

Step 3. Increase the selected index by exactly one and check the weight W^c of this potential child structure I_j^c.

If the weight W^c of this potential child structure I_j^c is greater than the current W^u, then go to Step 4.

Compute the relaxed weight R^c of the potential child structure. If the relaxed weight R^c is greater than the current W^u, then go to Step 4.

If there is a child node such that $P^0 = 0$ and $W^c < W^u$, then $W^u = W^c$, $I_j^u = I_j^c$ and $S^c = 1$, otherwise $S^c = 0$.

Let $fn = fn + 1$ and $N^{fn} = \left\{I_j^c, W^c, P^c, R^c, S^c\right\}$.

Step 4. Select the next cross-section index from index set I_j^{bn} which does not reach the maximal index. If there is no such an index, then go to Step 2, else go to Step 3.

6. Examples

The proposed method has been tested for several problems appeared in the literature [1]-[8]. The discrete minimum weight design is formulated in terms of member cross-sectional areas, member stresses and nodal displacements using geometrically nonlinear, large deflection structural model. According to the comparison examples stability constraints were not considered. In both problems presented here no design variable linking is present.

Problem 1: Design of a 10-bar cantilever truss

The results of the proposed method are compared to sequential and enumeration methods published by Gutkowski and Zawidzka [4]. The geometry and nodal coordinates are presented in Figure 1 and Figure 2. The related design variables may be found in the quoted literature [3], [4], and [8]. In this paper, only the non-convex ten-bar truss problem is presented. The initial, unfeasible structure is obtained by assigning to all members the smallest cross-sectional area from the catalog (see Figure 1). The truss is subject to the given 10^5 lb external load. The modulus of elasticity is 10^7 lb, and the material density $\rho = 0.1\ \ lb/in^3$ The displacement limit is $u_{max} = 2\ \ in$ along each degree of freedom. The stress constraint imposed on all members is $s_{max} = 2500\ \ psi$. The cross-sectional areas $A_i,\ \ i = 1,2,\dots,J$, as design variables are selected first time from the following set of ten available values of the catalog 1: $\{36.;27.0;19.0;12.0;7.0;4.0;2.0;1.0;0.5;0.1\}$, and from the set of thirteen available values of the catalog 2: $\{45.0;35.0;30.0;25.0;20.0;18.0;15.0;12.0;8.0;5.0;2.0;1.0;0.1\}$ in the second example. Results for both cases are presented in Tables 1 and 2.

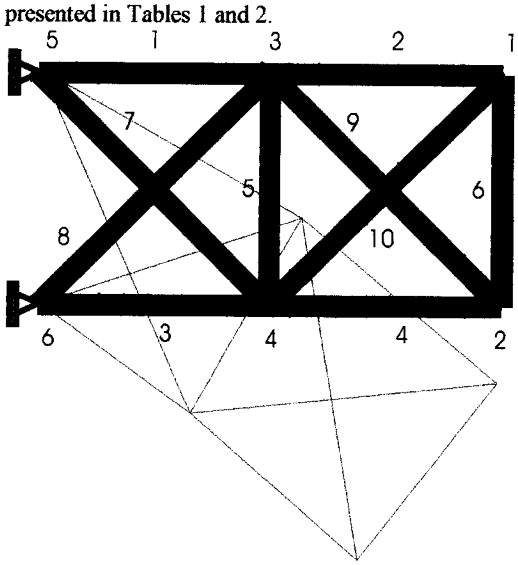

Figure 1.

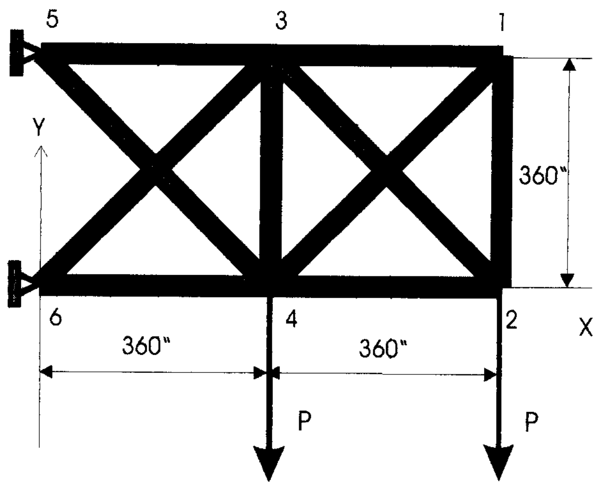

Figure 2

Present study:

variables	continuous solution	proposed method
1	30.4015	36.0
2	0.1	0.1
3	23.1041	27.0
4	15.2160	19.0
5	0.1	0.1
6	0.6623	0.5
7	7.5049	7.0
8	20.9631	19.0
9	21.5409	19.0
10	0.1	0.1
weight (lb)	5056.15	5273.32*

Gutkowski –Zawidzka [4]:

continuous solution	sequential algorithm	enumeration algorithm
30.031	36.0	36.0
0.1	0.5	0.5
23.274	27.0	27.0
15.286	19.0	19.0
0.1	0.5	0.5
0.5665	0.5	2.0
7.4683	7.0	7.0
21.198	27.0	19.0
21.618	19.0	19.0
0.1	0.5	0.1
5061.6	5477.77	5356.18*

Table 1: The optimum cross-sectional areas [in^2] using catalog 1.

*Note: The different weight is resulted from the usage of the geometrically nonlinear structural model instead of the linear structural model.

Present study:

variables	continuous solution	proposed method
1	30.4015	30.0
2	0.1	0.1
3	23.1041	25.0
4	15.2160	12.0
5	0.1	0.1
6	0.6623	0.1
7	7.5049	8,0
8	20.9631	20.0
9	21.5409	25.0
10	0.1	0.1
weight [lb]	5056.15	5158.61

Gutkowski and Zawidzka [4]:

continuous solution	sequential algorithm	enumeration algorithm
30.031	30.0	30.0
0.1	0.1	0.1
23.274	30.0	25.0
15.286	15.0	12.0
0.1	0.1	0.1
0.5665	0.1	1.0
7.4683	8.0	8.0
21.198	20.0	20.0
21.618	20.0	25.0
0.1	0.1	0.1
5061.6	5159.65	5158.61

Table 2: The optimum cross-sectional areas [in^2] using catalog 2.

<u>Problem 2. Design of a 25-bar truss</u>

The geometry of this well-known test problem for the initial and optimal design is presented in Figure 3 and Figure 4.

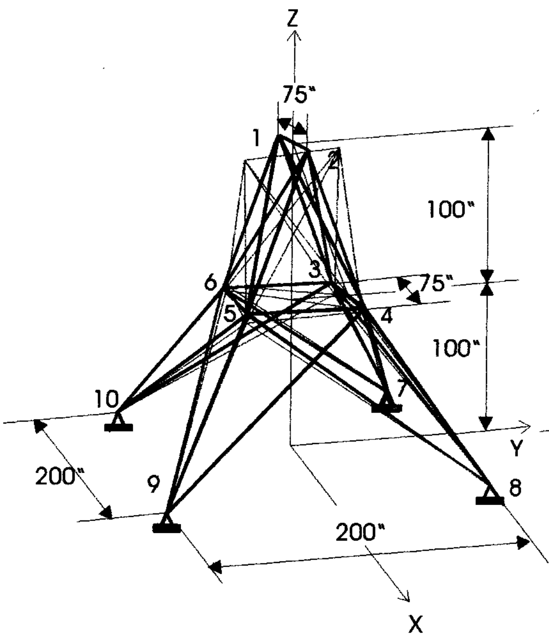

Figure 3.

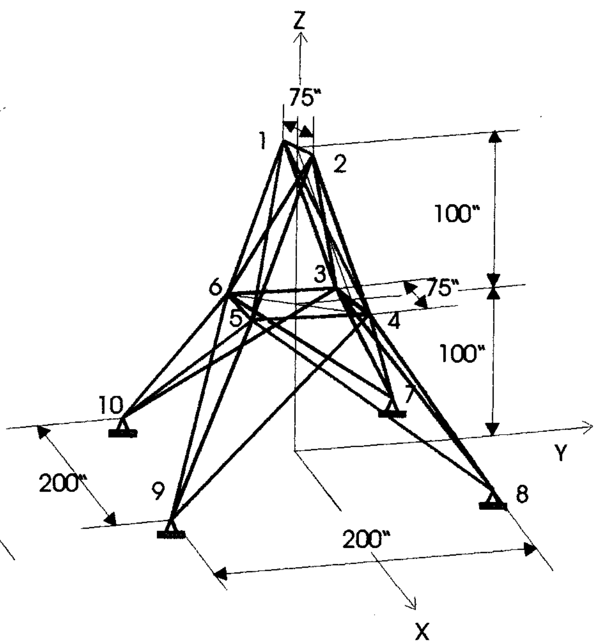

Figure 4.

The details of the optimal design problem, the stress and displacement limits, and the design variables for two applied load cases are illustrated by Allwood and Chung [1]. The authors presented an optimality criteria method for continuous design problem using a linear structural model. The 25-bar space truss has been analyzed and fully documented by other authors [3], [6]-[8] for mixed-discrete problem as well. In each case quitted, the design variables are linked to maintain symmetry.

In this paper only the load case II was considered. As no design variable linking is present, there are 25 design variables and 18 displacement variables originally. According to the symmetric property of the geometry the optimal design problem was extended with additional constraint of design variables lately. Our results obtained for the following selected discrete catalog values: $\{0.01;0.1;0.2;0.3;...;3.8\}$ are compared to pseudo-discrete rounding method published by Groenwold et al [3]. The large deflection in behavior of the initial structure obtained by assigning to all members the smallest catalog value is well demonstrated in Figure 3. According to the strong deflection limit in the comparison example [3], [8], in spite of the geometrically nonlinear model, the resulted structure is practically motionless (see Figure 4). Tabulated results for both cases, for continuous and mixed-discrete variables are presented in Table 5. This example demonstrates that the use of variable linking can lead to a worst design.

Node	Applied loads [kips]		
	X-force	Y-force	Z-force
1	-	20.00	-5.00
2	-	-20.00	5.00

Table 3: Applied loads of 25-bar truss

End nodes of the bars	Variables	Tensile stress limit [psi]	Compressive stress limit [psi]
1-2	1	40,000	-35,000
1-4; 2-3; 1-5; 2-6	2	40,000	-11,590
1-3; 1-6; 2-4; 2-6	3	40,000	-17,300
3-6; 4-5	4	40,000	-35,000
3-4; 5-6	5	40,000	35,000
3-10; 6-7; 4-9; 5-8	6	40,000	-6,760
4-7; 3-8; 5-10; 6-9	7	40,000	6,960
6-10; 3-7; 4-8; 5-9	8	40,000	-11,080

Table 4: Relationship between the truss-members and the variables.

	Present study:		Allwood and Chung [1]:	Groenwold et al [3]:
Variables	Continuous solution	Proposed method	Continuous solution	Pseudo-discrete rounding method
1	0.322891	0.30	0.01	0.01
2	1.999044	2.00	1.9845	2.10
3	2.697420	2.80	2.9973	3.00
4	0.039276	0.01	0.01	0.01
5	0.240454	0.01	0.01	0.01
6	0.344547	0.40	0.6841	0.60
7	1.99141	1.90	1.6773	1.60
8	0.045305	0.20	2.6609	2.80
weight [lb.]	397.43	403.897	545.168	547.04

Table 5: The optimum cross-sectional areas [in^2].

7. Conclusions

The benefit of the proposed method is its efficiency because of a strong reduction in number of the evaluated states. The method was developed for a geometrically nonlinear, large deflection structural model, which allows us to define the initial structure as an extremely infeasible one with very small cross-sectional areas. We have to note that only the full domain searching provides global optimal solution. The algorithm provides local optima, if the searching domain is restricted into a narrow range around of the relaxed solution. In this case, the efficacity of the algorithm highly depends on the "quality of the relaxed solution". A new element of the algorithm, that in the pruning process the relaxed weight computation can be replaced by a computationally cheaper and - in the general non-convex case - more safety lower bound estimation. Thus the proposed method involves an exterior point method to determine the relaxed solution of the design problem. In this method the optimal direction computation is formulated as an LP problem, the step-size is controlled by a maximal acceptable linearization-error. The success of the method for large-scale problems still needs verification.

8. Acknowledgments

This work was supported by the Hungarian National Science Foundation No. T 018324.
The first author would like to express her gratitude to Pauli Pedersen and Ole Sigmund for reading and commenting the manuscript. The support of the Danish Ministry of Education is appreciated.

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